НАУЧНИ ТРУДОВЕ
ТОМ LX
"ХРАНИТЕЛНА НАУКА, ТЕХНИКА И ТЕХНОЛОГИИ – 2013"
18-19 октомври 2013, Пловдив

SCIENTIFIC WORKS
VOLUME LX
„FOOD SCIENCE, ENGINEERING AND TECHNOLOGIES – 2013"
18-19 October 2013, Plovdiv

ORTHOGONAL MODELS FOR STEP RESPONSE WITH NOISE

Svetla Lekova[1], Vasil Drambalov[2]
[1]*University of Chemical Technology and Metallurgy, 8, Climent Ohridsky Blvd., Sofia 1756, BULGARIA, E-mail: sv_lekova@abv.bg*
[2]*Technical University–Sofia, Branch Plovdiv, E-mail: vs13@abv.bg*

Abstract
Various methods are known for the obtaining a model of the type of the transfer function from the database for step response. Implicit requirement of such methods is that the data of the characteristic to be relatively free from noise. In the present work, to obtain a model from step response with noise, the method of orthogonal decomposition applied directly to noisy step response. Although the filtering properties of orthogonal models in the literature traditionally analyzed objects/processes with negligible noise level.

For the step response it may use the integration by parts, so that the derivative instead of the step response to be held by the orthogonal functions. This way avoids the approximation of the derivative of the disturbance.

Good approximating properties of the process is concluded in its robustness to the overlay noise in output data and high reliability in both the time and frequency domain.

Keywords: *mathematical model; orthogonal functions; Kautz function.*

Introduction. Kautz filters.

The input signal significantly influences on the results of the identification. His choice depends on the available control and measuring devices, allowed disturbances, etc. The test signal should excites as much as possible own frequencies. The most popular one is a single step signal, the most simple and dominant in control systems. In the case of process without noise step response is a complete description of them. In the presence of noise it is usually apply different filtering methods, which could lead to inaccuracies.

On the other hand most existing methods for identification of the transfer function do not consider the time delay of the process (or insensitivity), or it is assumed that the delay time is accurately known. In the industrial processes time delay is important for obtaining a representative characteristic of control systems, hence the continuing interest in the identification of processes with a delay. In [3] were analyzed methods for approximation of unknown delay with a model based on rational transfer function (Laguerre formula), as well as with orthogonal Laguerre polynomials. Even when the system has big delay, these methods lead to an acceptable error of approximation.

The Kautz filters are proposed by Kautz in 1954. There were no computers at the time and only analytical solutions were possible. An important assumption when applying the functions of Laguerre or Kautz, is a quadratic integral value during the function to be limited (square integrable). In this case, the accuracy of the approximation is increased when the number of terms is increased and this ensures convergence.

There are three cases of Kautz framework that require different levels of complexity: with non-identical real poles, with non-identical complex poles, with a combination of real and complex poles. Relevant for the subject is the first case of distinct real poles [2]. Let $-\alpha_1/2, -\alpha_2/2, \ldots, -\alpha_N/2$ are the locations of the actual poles of Kautz framework (α_i is a positive real constant). The transfer function of the N-th Kautz filter is:

$$K_N(s) = q_N \frac{\prod_{i=1}^{N-1}(s-\alpha_i/2)}{\prod_{i=1}^{N-1}(s+\alpha_i/2)} \cdot \frac{1}{s+\alpha_N/2} \quad (1)$$

It is orthogonal to all preceding components. The normalization factor is $q_N = \sqrt{\alpha_N}$.

For this procedure, there is no limit if the pole is the same. Choosing the same pole the Laguerre functions are obtained. It is shown [] that applying Kautz functions (with different real poles) has significant advantages for processes developing with high speed in the beginning but dumping relatively slowly as well as for processes with time delay. When applying orthogonal Laguerre functions approximately the same precision is achieved with a much bigger number of coefficients.

Estimation of orthogonal models from step response with noise

Like the Laguerre model, the approximation of the impulse response $w(t)$ with model using Kautz framework is:

$$w(t) = \sum_{i=1}^{N} \beta_i k_i(\alpha_i t), \qquad (2)$$

Here $k_i(\alpha_i t)$ are Kautz functions. The Fourier coefficients are determined by

$$\beta_i = \int_0^{\infty} k_i(\alpha_i t)\, w(t)\, dt \qquad (3)$$

Let the step response of the system with a stable transfer function $W(s)$ is:

$$\tilde{h}(t) = h(t) + \xi(t) \qquad (4)$$

where $h(t)$ denotes the step response of the process, $\xi(t)$ - the output additive disturbance for which the assumption $|\xi(t)| < \infty$ is valid. One way to get a Kautz model, in particular – a Laguerre ones, from these data it is to get an estimate of the weighted response by differentiating the step response: $\tilde{w}(t) = d\tilde{h}(t)/dt$. However, the differentiation of the measured step response increases the effects of noise and leads to computational problems. Another approach is to use the integration by parts, so that the derivative instead of the step response to be held by the orthogonal functions. This avoids the approximation of the derivative of the disturbance.

Equation to evaluate the *i*-th Fourier coefficient of Kautz model of step response with noise $\tilde{h}(t)$ becomes [7]

$$\tilde{\beta}_i = \int_0^{\infty} \frac{d}{dt}\tilde{h}(t)\; k_i(\alpha_i t)\, dt = \qquad (5)$$
$$= \left[\tilde{h}(t) k_i(\alpha_i t)\right]_0^{\infty} - \int_0^{\infty} \tilde{h}(t)\; \dot{k}_i(\alpha_i t)\, dt$$

Such as for each $\alpha_i > 0$, $\lim_{t\to\infty} k_i(\alpha_i t) = 0$ and considering that $\tilde{h}(0) = 0$, the first member of the right-hand side of (5) is equal to zero. Therefore

$$\tilde{\beta}_i = -\int_0^{\infty} \tilde{h}(t)\; \dot{k}_i(\alpha_i t)\, dt \qquad (6)$$

For the approximated transfer function is followed:

$$\tilde{W}(\lambda, s) = \sum_{i=1}^{N} \tilde{\beta}_i(\lambda) K_i(\alpha_i s) \qquad (7)$$

where $K_i(\alpha_i s)$ are the Laplace images of the Kautz functions.

Implementation issues

In this section, several practical issues in implementation of the proposed algorithm are discussed.

Selection the location of the poles/ pole. In theory, the choice of the pole α slightly affects the existence and convergence patterns of Laguerre models. In practice, the incorrect selection of α leads to high-order models, in order to achieve the desired accuracy. The location of the pole α must be chosen so that for a given N the error in the given criteria is reduced. This approach is particularly effective if the poles of the system are in a relatively small area and the parameter α is selected in the neighborhood of the dominant pole of the system. Using Kautz functions desired precision of a lower-order model is achieved and the range is also broadened which allows a reasonable speed of convergence of the decomposition.

Error in step response. After obtaining the coefficients β_i, step response of the model can be calculated. Therefore, the integral square error between the measured step response $h(t)$ and step response of orthogonal model $h_{apr}(t)$ is:

$$E_{\text{step}} = \int_0^{Tm} [\tilde{h}(t) - h_{apr}(t)]^2\, dt \qquad (8)$$

where Tm is time greater than the time for establishing of process Tstep. (Tstep is defined as the time required for the process to reach ± 2% of steady state.) Based on this error function, the location of a pole/ poles can be chosen so that E_{step} is a minimum. Finding the best value of α (for a special case of the Laguerre functions), using criterion (8) can be solved by an unconditional one-dimensional optimization problem. But this is not applicable when using Kautz functions.

Model order. The order of the model N is also important for the accuracy of orthogonal model. Common practice in identification is to choose the order according to the type of change of the error function, for example E_{step}. The order N is chosen so that with its increase the error of model decreases insignificantly.

In determining the appropriate decision the rule of thumb is to choose $\boldsymbol{\alpha}$ such that the error between the step response of the sought-after pattern/ model and that taken from the real process [7] is minimized. In discrete form this error is:

$$sq = \frac{1}{M}\sum_{k=1}^{M}\left(h(kT_s) - h_{apr}(kT_s)\right)^2 \qquad (9)$$

where $h(kT_s)$ is the real output of the process in the step change of input, while the characteristic $h_{apr}(kT_s)$ is obtained from the model with the same input. (M - the number of discrete points in [0, Tm].)

Choice of Tm: The initial part of the step response contains frequency information about the process. Therefore it is pointless to use excessive data algorithm after steady state and so *Tm* must satisfy:

$$Tm = (1.2 \div 1.5)\,\text{Tstep} \qquad (10)$$

Choice of M: The number of calculations becomes a problem for many large points *M*. *M* is recommended to be about 200 [7].

Model order: In general, the order of the process is unknown. Most identification schemes assume that the model order is known. The proposed here approach does not follow such a strategy. If the *sq* error (9) is too large, the order of the evaluated model is increased by one. Furthermore, this approach makes no assumptions about the time delay.

Estimation of Kp: Common practice in the process control is the transient coefficient *Kp* be determined directly from the steady state as $Kp = [\,y(\infty) - y(0)\,] / h$ where h is the amplitude of input step signal. During the presentation of orthogonal functions knowledge of *Kp* has an important role in obtaining correction coefficients

$$K_N = \frac{K_p}{2\sum_{i=1}^{N}(-1)^{i-1}\beta_i/\sqrt{\alpha_i}} \qquad (11)$$

and eliminating the static error of the model.

Influence of steady state error: Common problem for all identification methods using step test signals is that low-frequency noise leads to errors of estimation. This error is reduced if the amplitude of the test signal or the signal / noise ratio is increased. If *Kp* is known the steady-state error can be removed and the proposed method can be applied to the modified curve of characteristic. After acceptance of N and calculation of the Fourier coefficients by (3) or (6) and K_N by (11) the coefficients $\{\beta_i\}$ can be adjusted:

$$\hat{\beta}_i = K_N\,\beta_i \qquad (12)$$

Error in frequency domain. For better evaluate the accuracy of the models, both errors in the time domain (8) and the errors of identification in the frequency domain are considered. In certain methods of identification, the characteristic in the time domain corresponds well to the process, but the frequency characteristic of the model may differs too much from the actual frequency response. To achieve good performance the error should be small and within the time and in the frequency domain. The model error in the frequency domain is estimated by the error in the worst conditions:

$$sw = \max_{w\in[0,w_\pi]}\left\{\left|\frac{\hat{W}(jw) - W(jw)}{W(jw)}\right| \times 100\%\right\} \qquad (13)$$

where $W(jw)$ is a frequency characteristic of the real process. Here, the frequency domain, which is considered [0, w_π], is determined by the frequency for which $\arg[\,W(jw_\pi)\,] = -\pi$.

Ratio noise/ signal. To show the robustness of the proposed method it is added a noise to the output of the process. In the context of the system identification, the noise/ signal ratio is determined by

$$\text{NSR} = \frac{\text{mean(abs(noise))}}{\text{mean(abs(signal))}} \qquad (14)$$

Simulation modeling of experimental step response

Example of application of the Kautz functions with real poles is in **processes / objects which may be regarded as the sum of first-order objects or the higher order with self-settling** with a large difference in the time constants and a pure delay. Processes with similar characteristics are the absorption/ desorption of wood fibrous sorbents for purification of water from oil contamination, and the purification of waters by heavy metals [4, 5] as well as the experimental curves of viscoelastic deformation discussed in [1, 6]. All these processes are characterized by high speed at the beginning, but relatively slowly reach the steady state.

Comparison of approximations with orthogonal Kautz and Laguerre models here addresses to the following test case:

$$W(s) = \frac{0.5\,exp(-0.25\,s)}{(1+s)(1+0.5\,s)} + \frac{0.5}{1+0.1\,s} \qquad (15)$$

The step response of the process are taken under 3 levels of noise: NSR = 0% , 10% and 30%. In accordance with the above section is selected: Tm = 8s, T_s = 1/30s, i.e. M = 241; w_{π} = 2.785. With the orthogonal approximations (model) of (15) we study the influence of the pure delay and noise on the scaling vector $\boldsymbol{\alpha}$, using the functions of Laguerre and Kautz. The results of the modeling are summarized in Table 1. Because of the significant delay, even when the poles of the Kautz filters are identical with the poles of the transfer function (15), namely, $\boldsymbol{\alpha}/2$ = [10 2 1], the error sq in step response is different from zero (st row in Table1). In approximation with orthogonal Laguerre functions, if the scaling factor α is chosen only considering the largest time - constant, then α must be in the interval [1.6, 5]. Due to the complexity of the transfer function (15), a higher value for α is preferable, which takes into account the steepness of the transition at the beginning of the characteristic. Most acceptable results have been obtained at α = 6, N = 5, NSR = 0%. In a significant noise and delay, the error of the model does not diminish with increasing its order, at least within certain limits. This follows from the fact that with the increase of the sequence number of the function Laguerre/ Kautz its overshooting increases, thus reducing the filtering properties. In the last two rows in Table 1 , the two values for sq correspond to the step response without noise and with noise (in formula (9)). On fig.1,2,3 are presented the step response approximations for these three discussed cases (at different level of noise) and $\boldsymbol{\alpha}$ = [25 6 6 6]. Good agreement of experimental and approximated step response confirms robustness of the method to both the choice of $\boldsymbol{\alpha}$ and the level of noise, which here is high enough - NSR = 30%.

Table 1.

NSR	$\boldsymbol{\alpha}$	β	*sq*	*sw*
0%	[20 4 2]	[1.1198 -0.1496 0.2444]	2.01 x10^{-5}	2.16%
	[19 4 2]	[1.1144 -0.0712 0.2822]	1.01 x10^{-5}	2.02%
	[17 6 3 3]	[0.1175 0.0449 1.3349 1.3324]	5.66x10^{-4}	4.49%
	[6 6 6 6 6]	[0.9999 0.3219 0.4615 0.0650 0.1258]	5.64x10^{-5}	3.81%
	[25 6 6 6]	[1.1104 -0.2329 0.1711 -0.1293]	9.09x10^{-5}	3.64%
10%	[25 6 6 6]	[1.0860 -0.2034 0.0030 -0.3846]	0.0099 0.0008	20.38%
30%	[25 6 6 6]	[1.1181 -0.1355 -0.0981 -0.5450]	0.0825 0.0011	39.54%

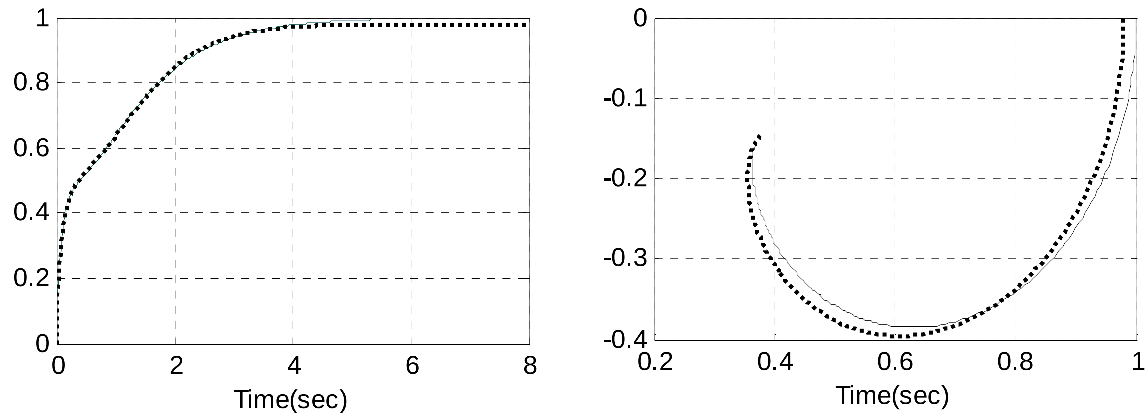

Fig.1. Orthogonal model, NSR = 0%

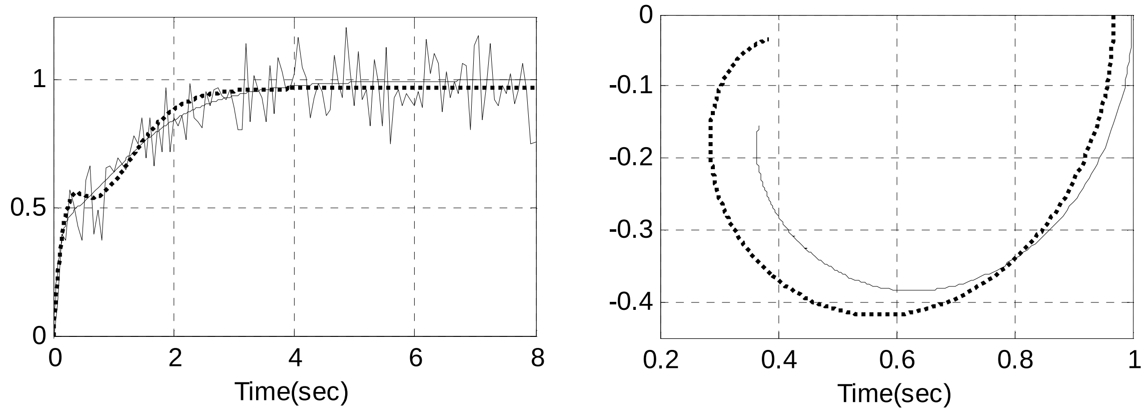

Fig.2. Orthogonal model, NSR = 10%

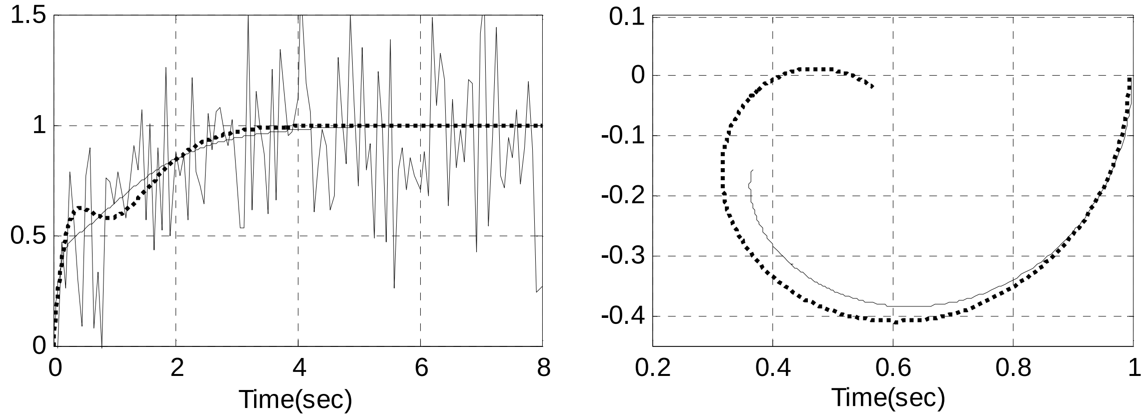

Fig.3. Orthogonal model, NSR = 30%

Conclusion

A. By comparing the approximation with Kautz and Laguerre functions convincingly are shown the characteristics and advantages of using Kautz functions. The following conclusions can be drawn:

- Kautz functions are applicable in case of objects with a large difference in time constants and pure delay. When using Laguerre functions approximately the same accuracy is obtained with a much larger number of coefficients.

- In the presence of pure delay it is necessary the poles of Kautz filters to be selected smaller than the poles of the transfer function.

- It is shown that the approximation with Kautz functions is robust to the choice of $\boldsymbol{\alpha}$ in a quite wide range, even if it is not necessary to cover the whole range of time constants/ poles.

- By comparing approximation with Kautz functions and with Laguerre functions the advantages of using Kautz functions are convincingly demonstrated.

B. Significant advantages of orthogonal models for step characteristics with noise are:

- do not differentiate step response;
- not to carry out its smooth;
- does not need inverse matrices (unlike most approximation algorithm) and waste related with this poor conditioning;
- in strength are all the advantages of using the orthogonal functions;
- It is needed a little a priori information about the order and time delay of the object. The method is applicable in a presence of pure delay and non-minimum phase systems, but does not apply if its step response does not start from zero.

References

[1] Aleksandrov, A., S. Lekova, M. Milenova. Generalized Model of Viscoelastic Deformation. Int. J. of Pure and Appl.Mathematics. Vol.81, No4, 2012, 635-646.

[2] Deschrijver, D., T. Dhaene. Data-driven model order reduction using orthonormal vector fitting. Model Order Reduction: Theory, Research Aspects and Applications, 13, No. 3 (2008), 341-359.

[3] Lekova, S. DYNAMIC MODELS OF OBJECTS

WITH TIME DELAY. Journal of the Technical University Sofia, branch Plovdiv, "Fundamental Sciences and Applications", Vol. 14, 2009, pp. 161-168.

[4] Lekova, S. Modeling processes to purify water from heavy metals. J. of Ecology and Industry, Volume 8, 2006, № 1-2, p.143-144.

[5] Lekova, S., S. Nenkova. Optimization of wood fiber sorbents to purify water from oil pollution. J. of Ecology and Industry, Volume 8, 2006, № 1-2, p.141-142.

[6] Lekova, S., A. Aleksandrov, M. Milenova, Tz. Tzolov. Orthogonal dynamic model of viscoelastic deformation under variable concentration and loading. Int. Scientific Conf. on Mechanics (MECH2012), Institute of Mechanics – BAS, Nov. 19-22, 2012.

[7] Wang, L., W. Cluett. From Plant Data to Process Control: Ideas for Process Identification and PID Design. Taylor and Francis, London, 2000.